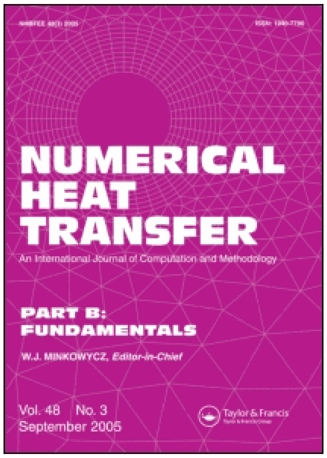

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An Efficient Segregated Algorithm for Incompressible Fluid Flow and Heat Transfer Problems - IDEAL (Inner Doubly Iterative Efficient Algorithm for Linked Equations) Part II: Application Examples

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AN EFFICIENT SEGREGATED ALGORITHM FOR INCOMPRESSIBLE FLUID FLOW AND HEAT TRANSFER PROBLEMS—IDEAL (INNER DOUBLY ITERATIVE EFFICIENT ALGORITHM FOR LINKED EQUATIONS) PART II: APPLICATION EXAMPLES

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In this article, comprehensive comparisons are made between the SIMPLER and IDEAL algorithms for four application examples. It is found that the IDEAL algorithm is efficient and stable not only for the simple, low-Re/Ra or coarse-mesh flow cases, but also for the complex, high-Re/Ra or fine-mesh flow cases. For the low-Re/Ra, coarse-mesh flow cases, the ratio of CPU time of IDEAL to that of SIMPLER ranges from 0.029 to 0.7. For the high-Re/Ra, fine-mesh flow cases, the IDEAL algorithm can obtain convergent results but the SIMPLER algorithm cannot, even though the underrelaxation factors are adjusted.

1. INTRODUCTION

In Part I of this article [1], an efficient segregated solution procedure for incompressible fluid flow and heat transfer problems was proposed. The new algorithm is called IDEAL (Inner Doubly Iterative Efficient Algorithm for Linked Equations). In the new algorithm the pressure equation is solved by inner doubly iterative processes. The first inner iteration process for the pressure equation almost completely overcomes the first approximation in the SIMPLE algorithm (inconsistency in initial fields of pressure and velocity); the second inner iteration process almost completely overcomes the second approximation in the SIMPLE algorithm (incompleteness in discretized pressure equation). Thus the coupling between velocity and pressure is almost fully guaranteed, greatly enhancing the convergence rate and stability of the iteration process not only for the simple, low-Re/Ra or coarse-mesh flow cases, but also for the complex, high-Re/Ra or fine-mesh flow cases. In the IDEAL algorithm the inner doubly iterative times N1, N2 are adjusted to control the convergence

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NOMENCLATURE

a	coefficient in the discretized equation; thermal diffusivity
A	surface area
b	constant term in the discretized equation
g	gravitational acceleration
N1, N2	inner doubly iterative times
p	pressure
q_m	reference mass flow rate
Ra	Rayleigh number
Re	Reynolds number
$\mathrm{Rs}_{\mathrm{Mass}}$	relative maximum mass residual
$\mathrm{Rs}_{U\mathrm{Mom}}$, $\mathrm{Rs}_{V\mathrm{Mom}}$	relative maximum u-, v-component momentum residuals
u, v	velocity component in x, y directions
U, V	nondimensional velocity component in X, Y directions
x, y	coordinates
X, Y	nondimensional coordinates
α	underrelaxation factor
β	expansion coefficient
ΔT	temperature difference
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
Subscripts	
e, w, n, s	cell surface
E, N, S, W, P	grid point
in	inlet
m	mean
nb	neighboring grid points
u, v	referring to u, v momentum equation
Superscripts	
0	previous iteration
$*$	intermediate value in iteration

of iteration process instead of the underrelaxation factors, which can be set as large as 0.9 ($\alpha_{u,v,p} = 0.9$). The big fixed underrelaxation factors guarantee a rather fast convergence rate for any flow case.

In this article, comprehensive comparisons are made between the SIMPLER and IDEAL algorithms for four two-dimensional problems of fluid flow and heat transfer:

1. Lid-driven cavity flow in a square cavity
2. Natural convection in a square cavity
3. Laminar fluid flow over a rectangular backward-facing step
4. Natural convection in a square cavity with an internal isolated vertical plate

All four problems are based on the following assumptions: laminar, incompressible, steady state, and constant fluid properties.

In the following, the comparison conditions and the convergence criterion are described first, followed by detailed presentations of computational results for the four examples. Finally, some conclusions are drawn.

2. NUMERICAL COMPARISON CONDITIONS

To make meaningful comparisons between the SIMPLER and IDEAL algorithms, numerical compassion conditions should be specified. In our study these conditions include the following.

1. *Discretization scheme*. In our study, Re/Ra has large-scale variations. In order to guarantee both numerical stability and solution accuracy, the SGSD scheme

[2] is adopted, which is at least second-order accurate and absolutely stable. For stability of the iteration process, the deferred-correction method is adopted, which was proposed in [3] and latter enhanced in [4].

2. *Solution of the algebraic equations*. The algebraic equations are solved by the alternating direction implicit method (ADI) incorporating the block-correction technique [5].

3. *Underrelaxation factor*. In the IDEAL algorithm the underrelaxation factors remain constant for the problems studied ($\alpha_{u,v,p} = 0.9$). For the SIMPLER algorithm, the same values are adopted for the underrelaxation factors.

4. *Grid system*. For each problem the same uniform grid system is used for execution of both the SIMPLER and IDEAL algorithms. The details of each grid system will be presented individually.

5. *Convergence criterion*. The convergence criterion requires that both the relative maximum mass and the relative maximum u-, v-component momentum residuals are less than prespecified small values.

The relative maximum mass residual is expressed as

$$\mathrm{Rs}_{\mathrm{Mass}} = \frac{\mathrm{MAX}[|(\rho u^* A)_w - (\rho u^* A)_e + (\rho v^* A)_s - (\rho v^* A)_n|]}{q_m} \tag{1}$$

where u^*, v^* is the intermediate velocity of each iteration level and q_m is the reference mass flow rate. For the open system, we take the inlet mass flow rate as the reference mass flow rate. For the closed system, we make a numerical integration for the mass flow rate along any section in the field to obtain the reference mass flow rate [6].

The relative maximum u-, v-component momentum residuals are expressed as

$$\mathrm{Rs}_{U\mathrm{Mom}} = \frac{\mathrm{MAX}\left\{\left|a_e u_e^0 - \left[\sum_{\mathrm{nb}} a_{\mathrm{nb}} u_{\mathrm{nb}}^0 + b + A_e(p_P - p_E)\right]\right|\right\}}{\rho u_m^2} \tag{2}$$

$$\mathrm{Rs}_{V\mathrm{Mom}} = \frac{\mathrm{MAX}\left\{\left|a_n v_n^0 - \left[\sum_{\mathrm{nb}} a_{\mathrm{nb}} v_{\mathrm{nb}}^0 + b + A_n(p_P - p_N)\right]\right|\right\}}{\rho u_m^2} \tag{3}$$

where u^0, v^0 is the initial velocity of each iteration level and ρu_m^2 is the reference momentum. For the open system, we take the inlet momentum as the reference momentum. For the closed system, we make a numerical integration for the momentum along any section in the field to obtain the reference momentum [6].

3. NUMERICAL COMPARISONS BETWEEN SIMPLER AND IDEAL

3.1. Problem 1: Lid-Driven Cavity Flow in a Square Cavity

Lid-driven cavity flow in a square cavity has served over and over again as a benchmark problem for testing numerical procedures in computational fluid dynamics/numerical heat transfer CFD/NHT. For high values of Reynolds number Re, many solution procedures have been proposed [7–11]. Among these, Ghia et al. [11] adopted the vorticity-stream function method incorporating multigrid techniques

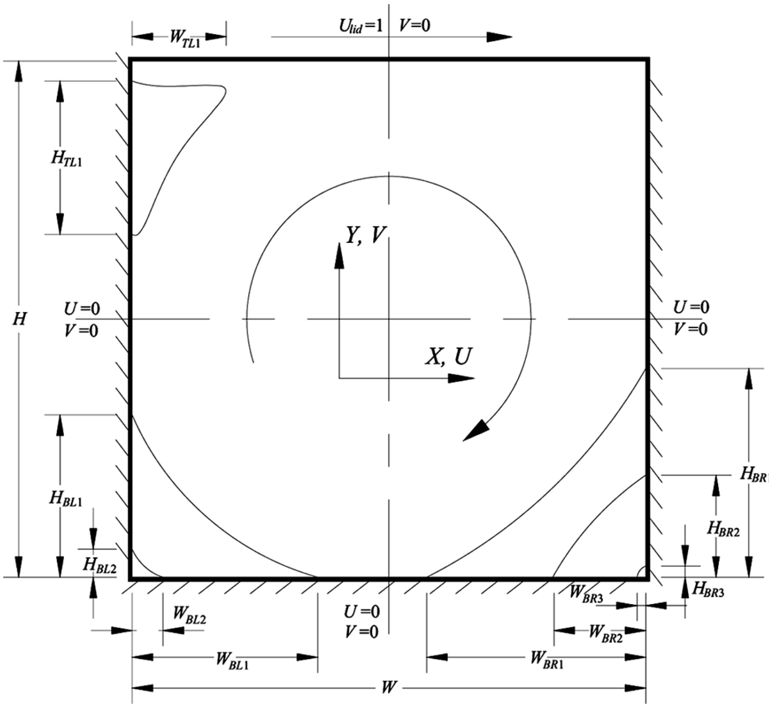


Figure 1. Lid-driven cavity flow configuration and boundary conditions.

to obtain solutions for high Reynolds numbers and fine mesh with Re = 10,000 and grid system = 257×257, and provided solutions in detail for Re = 100–10,000 and grid numbers = 129×129–257×257. Their results have long been regarded as the benchmark solutions in CFD/NHT communities. However, to the authors' knowledge, so far, pressure-correction methods have not been used successfully to obtain solutions for high-Re, fine-mesh flow, because the iteration process is prone to diverge very easily for these flow cases. In our study the IDEAL algorithm is used to obtain a solution for Re = 10,000 on a grid system of 260×260.

The configuration and boundary conditions for the lid-driven cavity flow are shown in Figure 1. Calculations are conducted for Re = 100–10,000 and grid numbers = 52×52–260×260, and the residuals $\mathrm{Rs}_{\mathrm{Mass}}$, $\mathrm{Rs}_{U\mathrm{Mom}}$, and $\mathrm{Rs}_{V\mathrm{Mom}}$ are all less than 10^{-7}. The Reynolds number is defined by

$$\mathrm{Re} = \frac{U_{\mathrm{lid}} W}{\nu} \tag{4}$$

where $W = H = 1$. Table 1 presents the convergence characteristics of both SIMPLER and IDEAL, including the CPU time, the iteration numbers, their ratio, and the inner doubly iterative times N1, N2 used in the IDEAL algorithm. As shown in Table 1, the IDEAL algorithm can converge for both the low-Re, coarse-mesh and the high-Re, fine-mesh flow cases, however, the SIMPLER algorithm can only converge for the low-Re, coarse-mesh flow cases.

Table 1. Convergence characteristics of both SIMPLER and IDEAL for Problem 1 (C, convergence; D, divergence)

Grid number	Comparison terms		Re 100	400	1,000	3,200	5,000	7,500	10,000
52 × 52	Time (s)	IDEAL	2.89	2.39	2.03	—	—	—	—
		SIMPLER	7.92	7.35	16.14	—	—	—	—
		Ratio	**0.365**	**0.325**	**0.126**	—	—	—	—
	Iteration number	IDEAL	234	184	163	—	—	—	—
		SIMPLER	1264	1152	2604	—	—	—	—
		Ratio	**0.185**	**0.160**	**0.063**	—	—	—	—
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	—	—	—	—
82 × 82	Time (s)	IDEAL	18.95	13.57	12.64	—	—	—	—
		SIMPLER	56.7	46.95	69.7	—	—	—	—
		Ratio	**0.334**	**0.289**	**0.181**	—	—	—	—
	Iteration number	IDEAL	499	356	334	—	—	—	—
		SIMPLER	3171	2623	3911	—	—	—	—
		Ratio	**0.157**	**0.136**	**0.085**	—	—	—	—
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	—	—	—	—
130 × 130	Time (s)	IDEAL	C	C	C	C	C	—	—
		SIMPLER	D	D	D	D	D	—	—
		Ratio	0	0	0	0	0	—	—
	Iteration number	IDEAL	C	C	C	C	C	—	—
		SIMPLER	D	D	D	D	D	—	—
		Ratio	0	0	0	0	0	—	—
	N1, N2 used in IDEAL		5, 5	5, 5	5, 5	5, 5	5, 5	—	—
260 × 260	Time (s)	IDEAL	C	C	C	C	C	C	C
		SIMPLER	D	D	D	D	D	D	D
		Ratio	0	0	0	0	0	0	0
	Iteration number	IDEAL	C	C	C	C	C	C	C
		SIMPLER	D	D	D	D	D	D	D
		Ratio	0	0	0	0	0	0	0
	N1, N2 used in IDEAL		10,10	10,10	10,10	10,10	10,10	10, 10	10, 10

The streamline contours for the lid-driven cavity flow with Re increasing from 1,000 to 10,000 from the IDEAL algorithm are shown in Figure 2 and are in excellent agreement with those reported by Ghia et al. [11]. In Figures 3–5, the velocity distributions along the two centerlines from the IDEAL algorithm are shown, and the benchmark solutions from Ghia et al. [11] are also presented. It can be seen that they are almost identical. A comprehensive survey of the properties of the vortices in the lid-driven cavity flow is provided in Table 2. As shown in Figure 1, the subscripts T, B, L, and R denote top, bottom, left, and right, respectively; the subscript numbers 1, 2, and 3 denote the hierarchy of these secondary vortices. For example, W_{BR2} stands for the width size of the second in the sequence of secondary vortices that occurs at the bottom right corner of the cavity. Comparing the sizes of the vortices from both the IDEAL algorithm and those of Ghia et al. [11], we find that they agree well with each other. From these comprehensive comparisons with benchmark solutions, it can be concluded that the IDEAL algorithm can obtain accurate solutions for both the low-Re, coarse-mesh and the high-Re, fine-mesh flow cases.

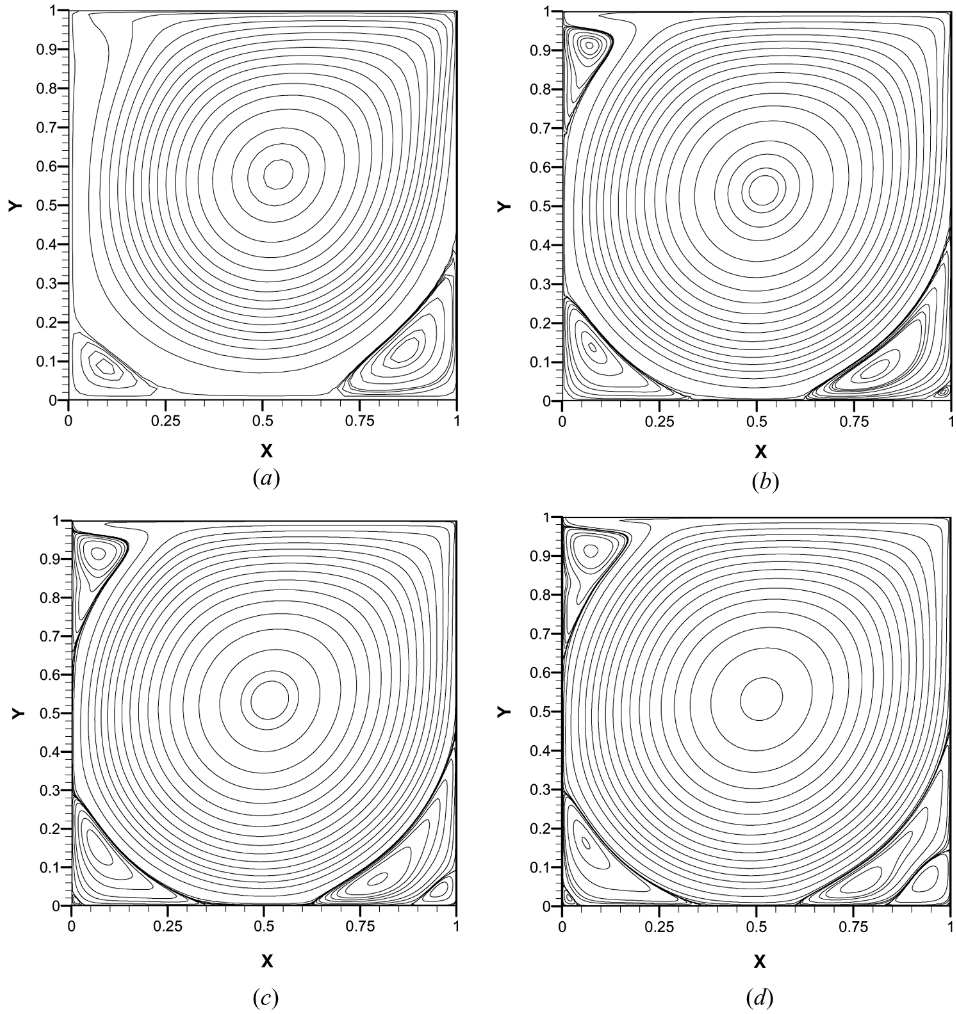


Figure 2. Streamline patterns from the IDEAL algorithm for Problem 1: (*a*) Re = 1,000, grid number = 52 × 52; (*b*) Re = 5,000, grid number = 130 × 130; (*c*) Re = 7,500, grid number = 260 × 260; (*d*) Re = 10,000, grid number = 260 × 260.

For the low-Re, coarse-mesh flow cases, the convergence rate and stability of both the IDEAL and SIMPLER algorithms are compared in detail. Figure 6 shows a plot of the ratios of CPU time and iteration numbers versus Re for different grid numbers. It can be seen that the ratios decrease with increase of Re or grid number, except for the point Re = 1,000, where the ratios increase with increase of grid number. The ratio of iteration numbers of IDEAL over that of SIMPLER ranges from 0.063 to 0.185 and the ratio of the CPU time ranges from 0.126 to 0.365. Figure 7 shows the convergence history of both the IDEAL and SIMPLER algorithms. As shown in Figure 7, the IDEAL algorithm is much more stable than SIMPLER. For the high-Re, fine-mesh flow cases, the IDEAL

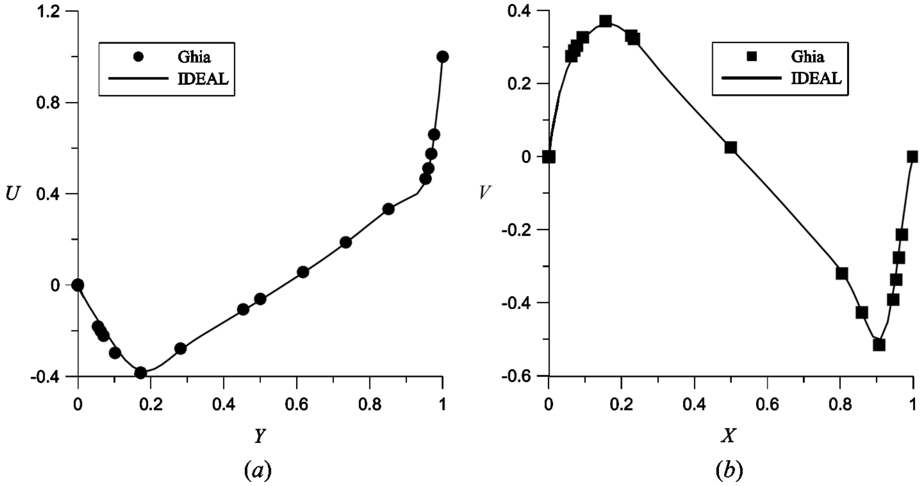


Figure 3. Predicted velocity distributions for Re = 1,000 and grid number = 52×52 in Problem 1: (*a*) *U*-component distribution along $X = 0.5$; (*b*) *V*-component distribution along $Y = 0.5$.

algorithm remains stable during the iteration process (shown in Figure 8), while SIMPLER diverges.

3.2. Problem 2: Natural Convection in a Square Cavity

The square cavity has two adiabatic walls (top and bottom), with its two vertical walls being maintained at constant but different temperatures, as shown in Figure 9. Calculations are conducted for Ra = 10^4–10^8 and grid numbers = 62×62–102×102.

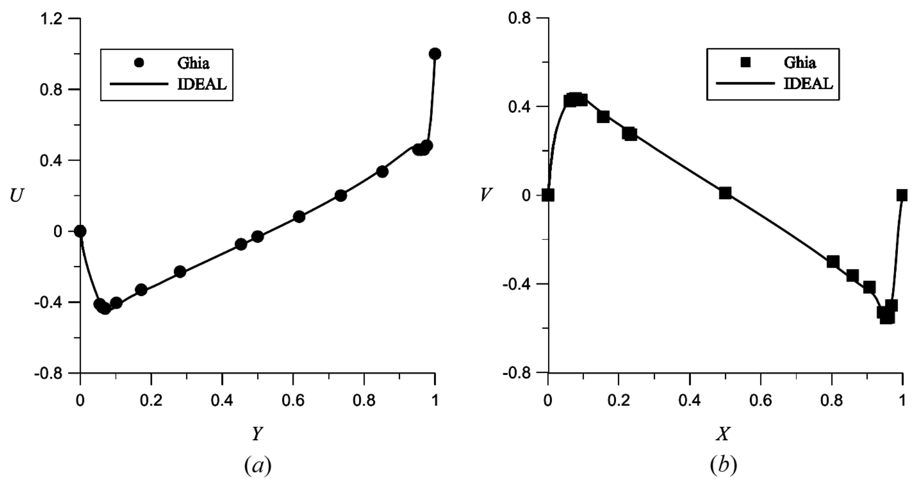


Figure 4. Predicted velocity distributions for Re = 5,000 and grid number = 130×130 in Problem 1: (*a*) *U*-component distribution along $X = 0.5$; (*b*) *V*-component distribution along $Y = 0.5$.

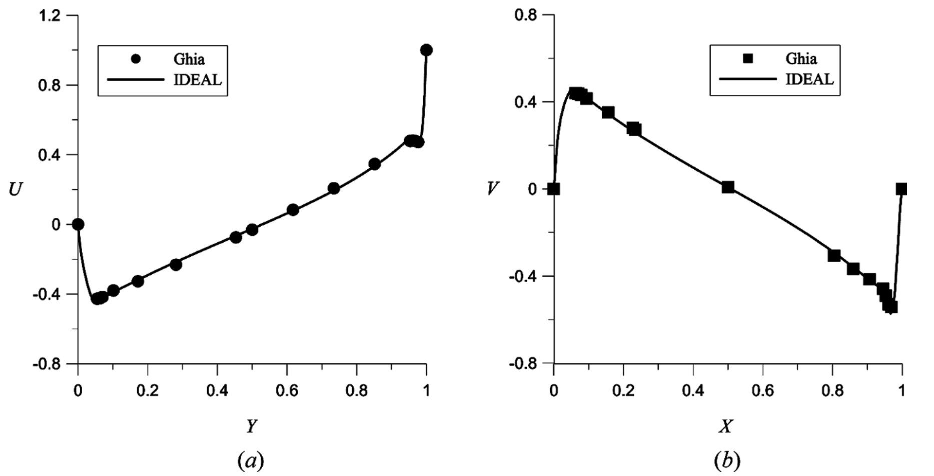


Figure 5. Predicted velocity distributions for Re = 10,000 and grid number = 260 × 260 in Problem 1: (*a*) *U*-component distribution along $X = 0.5$; (*b*) *V*-component distribution along $Y = 0.5$.

The residuals Rs_{Mass}, Rs_{UMom}, and Rs_{VMom} are all less than 5.0×10^{-7}. The Rayleigh number is defined by

$$\mathrm{Ra} = \frac{\rho g \beta W^3 \Delta T}{a\mu} \tag{5}$$

where $W = H = 1$. Table 3 presents the convergence characteristics of both SIMPLER and IDEAL. As shown in Table 3, the IDEAL algorithm converges for both the low-Ra and high-Ra flow cases; however, the SIMPLER algorithm converges only for the low-Ra flow cases.

The results of calculations with the IDEAL algorithm are presented in graphical form in Figure 10. Streamlines and isotherms are shown for $\mathrm{Ra} = 10^6$ and $\mathrm{Ra} = 10^8$. For $\mathrm{Ra} = 10^6$ the results agree very well with those reported by Barakos

Table 2. Properties of the vortices for Problem 1

		Property											
Re	Reference	W_{TL1}	H_{TL1}	W_{BL1}	H_{BL1}	W_{BL2}	H_{BL2}	W_{BR1}	H_{BR1}	W_{BR2}	H_{BR2}	W_{BR3}	H_{BR3}
1,000	Ghia [11]	—	—	0.219	0.168	—	—	0.303	0.354	—	—	—	—
	IDEAL	—	—	0.210	0.170	—	—	0.300	0.360	—	—	—	—
5,000	Ghia [11]	0.121	0.269	0.318	0.264	0.016	0.042	0.357	0.418	0.053	0.042	—	—
	IDEAL	0.123	0.272	0.320	0.264	0.018	0.044	0.352	0.410	0.058	0.046	—	—
7,500	Ghia [11]	0.145	0.299	0.334	0.279	0.023	0.025	0.378	0.438	0.127	0.094	0.004	0.004
	IDEAL	0.146	0.307	0.335	0.285	0.025	0.029	0.374	0.440	0.120	0.094	0.005	0.005
10,000	Ghia [11]	0.159	0.320	0.344	0.289	0.035	0.044	0.391	0.449	0.171	0.137	0.004	0.004
	IDEAL	0.165	0.328	0.346	0.292	0.038	0.048	0.394	0.455	0.168	0.143	0.006	0.006

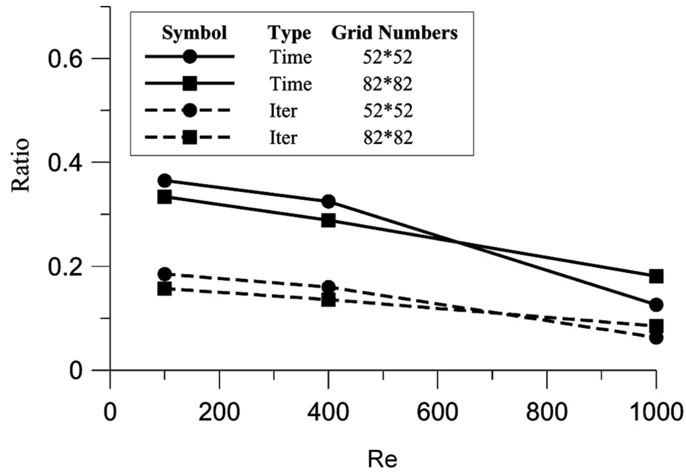


Figure 6. Ratios of CPU time and iteration numbers for Problem 1.

and Mitsoulis [12]; however, for $Ra = 10^8$ there are some differences between this work and that of Barakos and Mitsoulis [12]. In our study the second-order scheme (SCSG) is adopted, which is more accurate than the hybrid upwind differencing

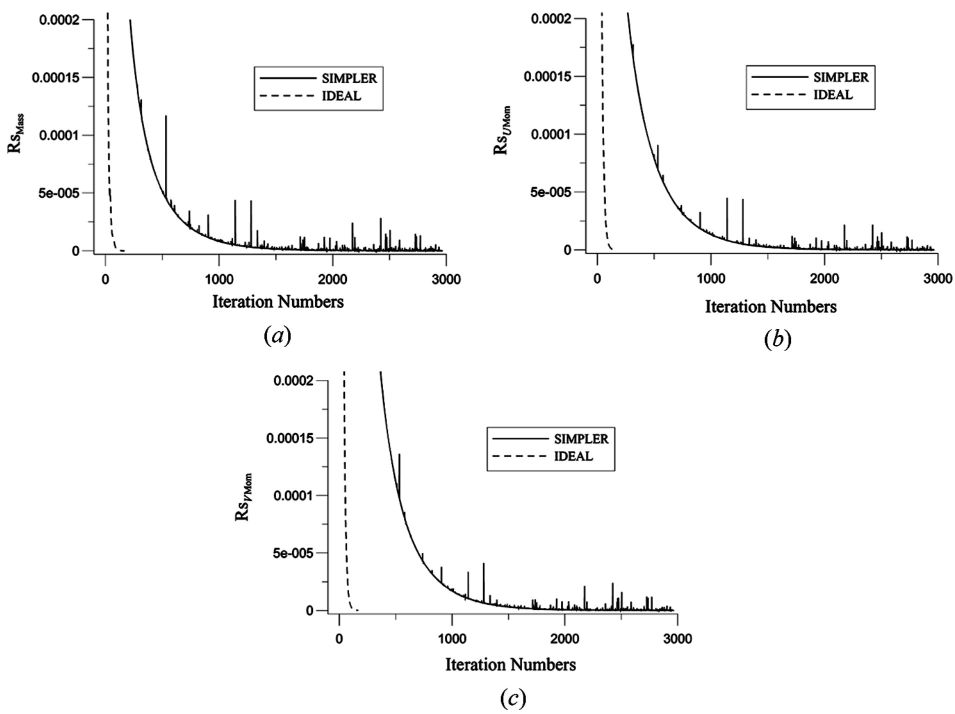


Figure 7. Convergence history of both the IDEAL and SIMPLER algorithms for $Re = 1{,}000$, grid number $= 52 \times 52$ for Problem 1: (*a*) mass residual; (*b*) *U*-component momentum residual; and (*c*) *V*-component momentum residual.

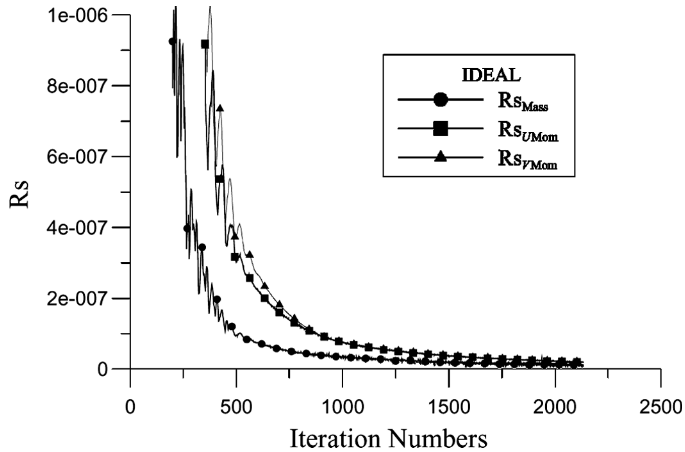


Figure 8. Convergence history of the IDEAL algorithm for Re = 10,000, grid number = 260 × 260 for Problem 1.

scheme (HDS) used by Barakos and Mitsoulis [12]. In Table 4, a comparison is given between the solutions from the IDEAL algorithm and the results from [12]. The comparison concerns the mean Nusselt numbers Nu. There is excellent agreement between the present results with the solutions by Barakos and Mitsoulis [12] for all values of Ra, except for the point Ra = 10^8, where there is some difference. It is believed that this difference is caused mainly by the different discretization schemes.

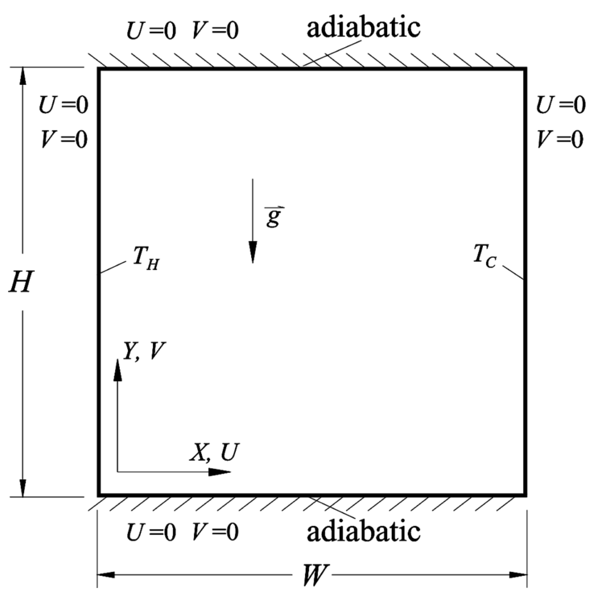


Figure 9. Scheme and boundary conditions for Problem 2.

Table 3. Convergence characteristics of both SIMPLER and IDEAL for Problem 2 (C, convergence; D, divergence)

Grid number	Comparison terms		Ra 10^4	10^5	10^6	10^7	10^8
62×62	Time (s)	IDEAL	16.06	11.8	10.23	—	—
		SIMPLER	22.93	25.59	24.07	—	—
		Ratio	**0.700**	**0.461**	**0.425**	—	—
	Iteration number	IDEAL	740	540	468	—	—
		SIMPLER	2009	2072	2083	—	—
		Ratio	**0.368**	**0.261**	**0.225**	—	—
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	—	—
82×82	Time (s)	IDEAL	49.01	33.91	27.34	—	—
		SIMPLER	71.78	74.03	74.34	—	—
		Ratio	**0.683**	**0.458**	**0.368**	—	—
	Iteration number	IDEAL	1185	837	657	—	—
		SIMPLER	3323	3333	3430	—	—
		Ratio	**0.357**	**0.251**	**0.192**	—	—
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	—	—
102×102	Time (s)	IDEAL	110.8	77.1	57.28	C	C
		SIMPLER	179.64	170.1	179.82	D	D
		Ratio	**0.617**	**0.453**	**0.319**	**0**	**0**
	Iteration number	IDEAL	1692	1179	879	C	C
		SIMPLER	5144	4838	5137	D	D
		Ratio	**0.329**	**0.244**	**0.171**	**0**	**0**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	4, 4	10,10

For the low-Ra flow cases, the convergence rate and stability of both the IDEAL and SIMPLER algorithms are compared. Figure 11 shows a plot of the ratios of CPU time and iteration numbers versus Ra for different grid numbers. It can be seen that the ratios decrease with increase of Ra or grid number. The ratio of iteration numbers varies from 0.171 to 0.368 and the ratio of CPU time ranges from 0.319 to 0.700. Figure 12 shows the convergence history of both the IDEAL and SIMPLER algorithms. As shown in Figure 12, the IDEAL algorithm is much more stable than SIMPLER. For the high-Ra flow cases, the IDEAL algorithm also remains stable during the iteration process (shown in Figure 13).

3.3. Problem 3: Laminar Fluid Flow over a Rectangular Backward-Facing Step

The scheme and boundary conditions are shown in Figure 14. Computations are conducted for Re = 100–500 and grid numbers = 62×32 and 122×50. The residuals Rs_{Mass}, $Rs_{U\mathrm{Mom}}$, and $Rs_{V\mathrm{Mom}}$ are all less than 5.0×10^{-7}. The geometric parameters are the same as those reported by Kondoh et al. [13]: $H_2/H_1 = 2$, $L_1/H_1 = 5$, $L_2/H_1 = 30$. The inlet velocity distribution is fully developed:

$$X = 0 \qquad 1 < Y < \frac{H_1 + H_2}{H_1} \qquad U_{\mathrm{in}} = 1.5\left\{1 - \left[\frac{Y - 0.5(H_2/H_1) - 1}{0.5(H_2/H_1)}\right]^2\right\} \tag{6}$$

where X, Y are the nondimensional coordinates.

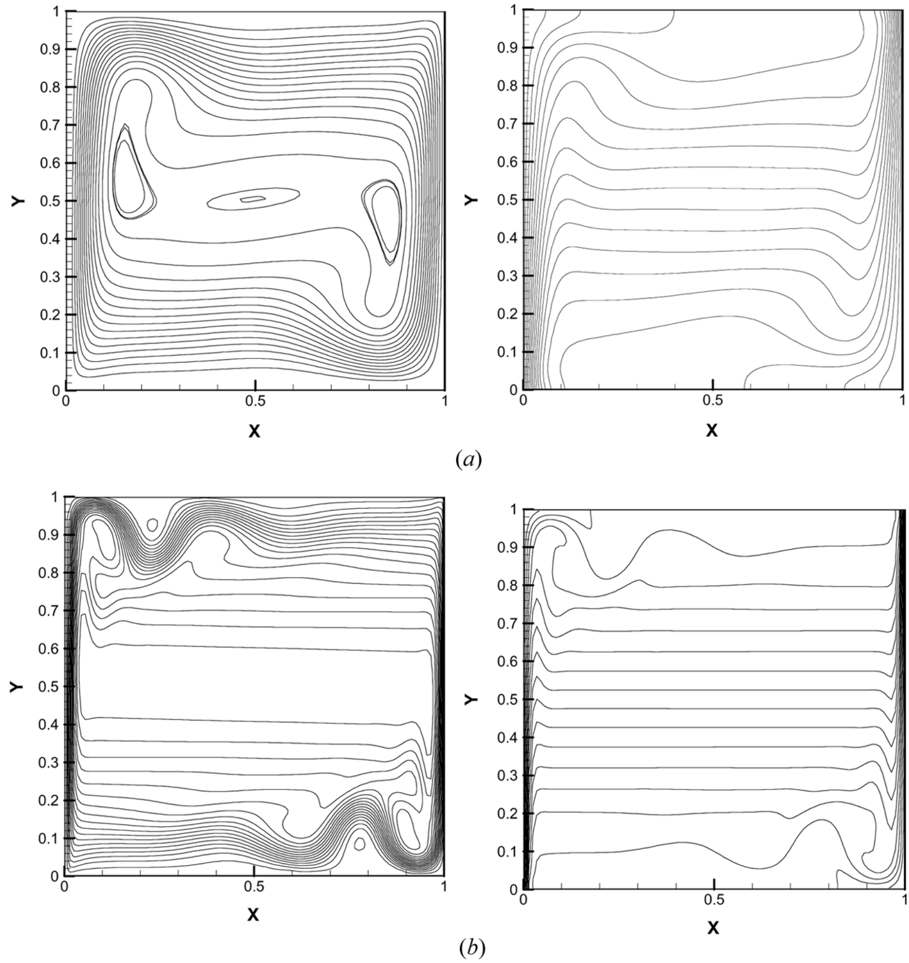


Figure 10. Streamline (left) and isotherm (right) patterns using the IDEAL algorithm for Problem 2: (*a*) $\mathrm{Ra} = 10^6$, grid number $= 102 \times 102$; (*b*) $\mathrm{Ra} = 10^8$, grid number $= 102 \times 102$.

The Reynolds number is defined as

$$\mathrm{Re} = \frac{U_m H_1}{\nu} \tag{7}$$

where U_m is the mean velocity at the inlet section. In the domain $0 < X < L_1/H_1$, $0 < Y < 1$, the solid region is treated with the domain extension method [6].

Table 4. Comparison of solutions with previous works for different Ra values in Problem 2

Ra	10^4	10^5	10^6	10^7	10^8
Barakos [12]	2.245	4.510	8.806	—	30.1
IDEAL	2.250	4.544	8.915	17.09	33.18

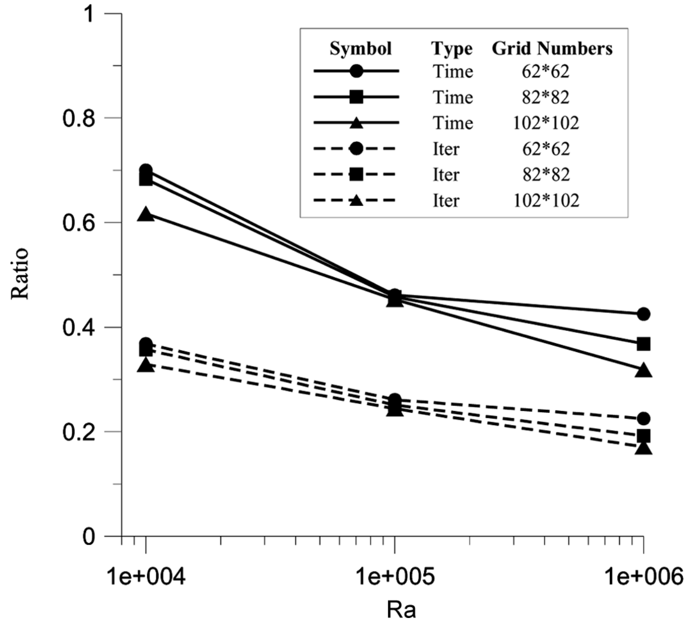


Figure 11. Ratios of CPU time and iteration numbers for Problem 2.

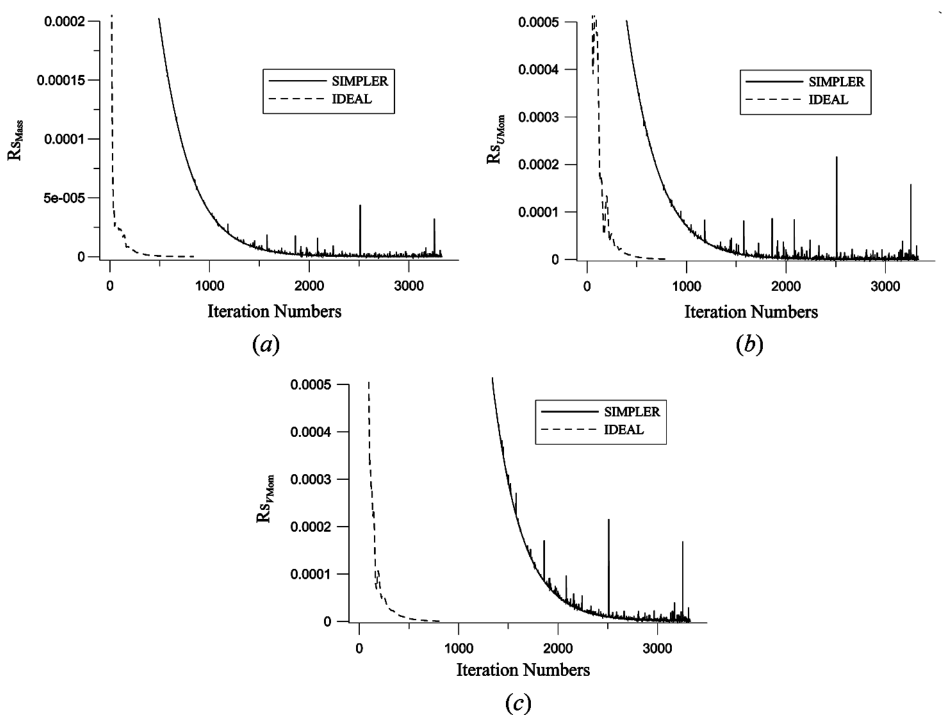


Figure 12. Convergence history of both the IDEAL and SIMPLER algorithms for $Ra = 10^5$, grid number $= 82 \times 82$ for Problem 2: (*a*) mass residual; (*b*) *U*-component momentum residual; (*c*) *V*-component momentum residual.

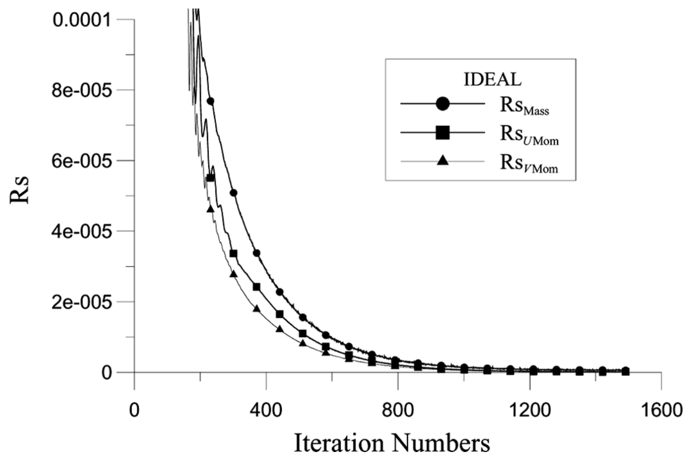


Figure 13. Convergence history of the IDEAL algorithm for $Ra = 10^7$, grid number $= 102 \times 102$ for Problem 2.

Table 5 presents the convergence characteristics for both SIMPLER and IDEAL. As only the laminar flow is to be considered, the Reynolds number is examined up to 500. A plot of the ratios of CPU time and iteration numbers versus Re for different grid numbers is shown in Figure 15. It can be seen that the ratios decrease with increase of Re or grid number, except for the point $Re = 500$, where the ratios increase with increase of Re. The ratio of iteration numbers varies from 0.014 to 0.029 and the ratio of the CPU time varies from 0.029 to 0.060. The IDEAL algorithm shows very large enhancement of the convergence rate in this problem. The convergence history of both the IDEAL and SIMPLER algorithms is shown in Figure 16. As shown in the figure, the IDEAL algorithm is much more stable than SIMPLER.

3.4. Problem 4: Natural Convection in a Square Cavity with an Internal Isolated Vertical Plate

The square cavity has two adiabatic walls (top and bottom), with its two vertical walls being maintained at a constant temperature. There is an internal isolated

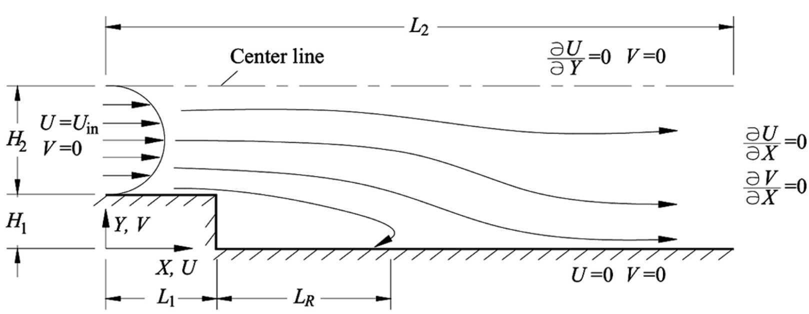


Figure 14. Scheme and boundary conditions for Problem 3.

Table 5. Convergence characteristics of both SIMPLER and IDEAL for Problem 3

Grid number	Comparison terms		Re 100	Re 300	Re 500
62×32	Time (s)	IDEAL	2.2	1.8	2.84
		SIMPLER	43.83	46.47	47.64
		Ratio	**0.050**	**0.039**	**0.060**
	Iteration number	IDEAL	247	206	317
		SIMPLER	10015	10648	10885
		Ratio	**0.025**	**0.019**	**0.029**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4
122×50	Time (s)	IDEAL	13.78	11.4	20.2
		SIMPLER	382.3	394.92	380.75
		Ratio	**0.036**	**0.029**	**0.053**
	Iteration number	IDEAL	437	352	610
		SIMPLER	24220	25023	24035
		Ratio	**0.018**	**0.014**	**0.025**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4

vertical plate which is maintained at another constant temperature, as shown in Figure 17. Calculations are conducted for $Ra = 10^4$–10^7 and grid numbers = 82×82–162×162. The residual, Rs_{Mass}, is less than 5.0×10^{-8}. The isolated island is dealt with by the method proposed by Yang and Tao [14]. The Rayleigh number

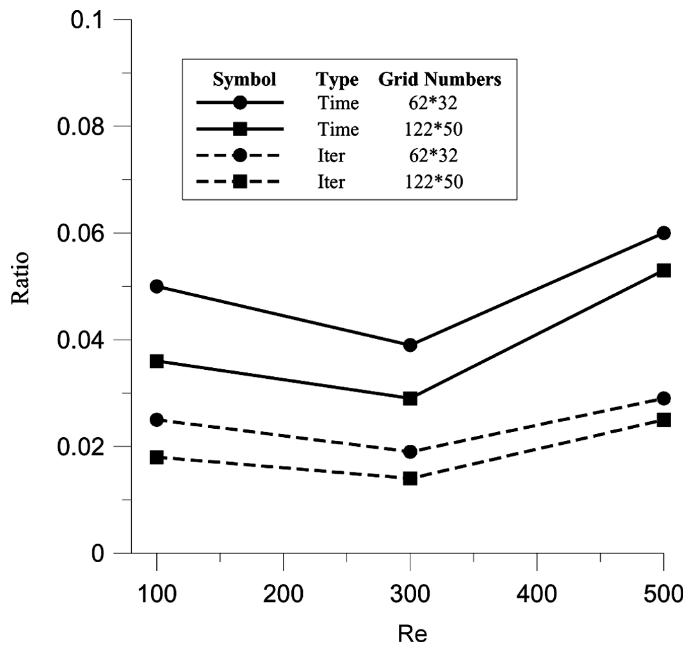
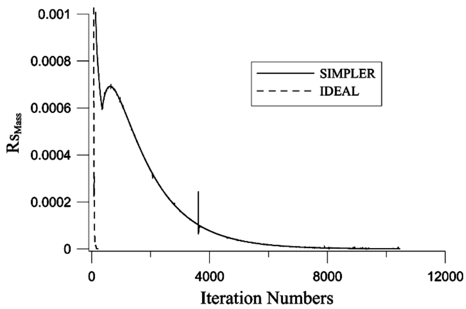
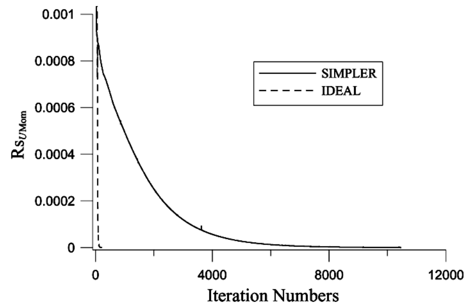


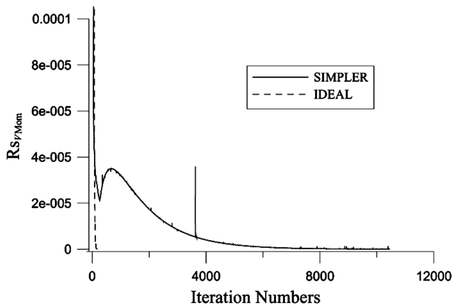
Figure 15. Ratios of CPU time and iteration numbers for Problem 3.



(*a*)



(*b*)



(*c*)

Figure 16. Convergence history of both the IDEAL and SIMPLER algorithms for $\mathrm{Re} = 300$, grid number $= 62 \times 32$ for problem 3: (*a*) mass residual; (*b*) *U*-component momentum residual; (*c*) *V*-component momentum residual.

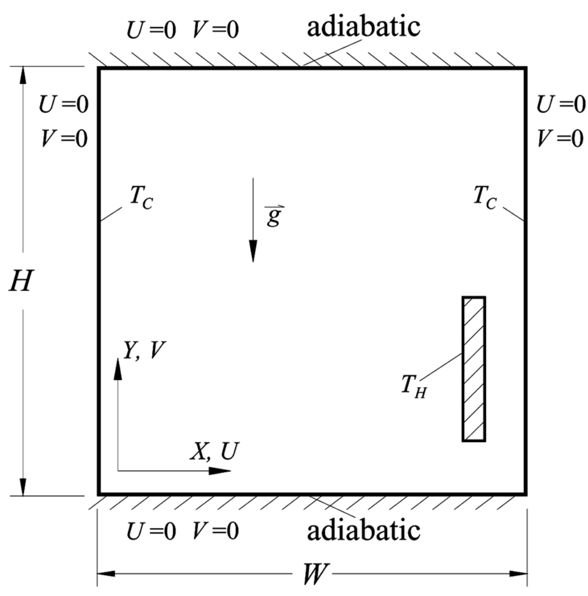


Figure 17. Scheme and boundary conditions for Problem 4.

is defined by

$$\mathrm{Ra} = \frac{\rho g \beta W^3 \Delta T}{a\mu} \tag{8}$$

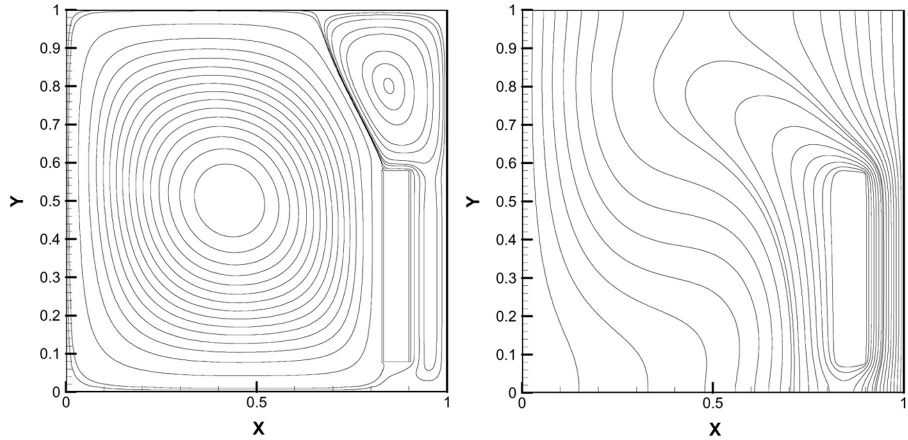
where $W = H = 1$. Table 6 presents the convergence characteristics of both SIMPLER and IDEAL. As shown in Table 6, the IDEAL algorithm can converge for both the low-Ra, coarse-mesh and the high-Ra, fine-mesh flow cases, however, the SIMPLER algorithm can converge only for the low-Ra, coarse-mesh flow cases.

The results of calculations with the IDEAL algorithm are presented in graphical form in Figure 18. Streamlines and isotherms are shown for $\mathrm{Ra} = 10^4$ and $\mathrm{Ra} = 10^7$. For $\mathrm{Ra} = 10^4$ the result agrees very well with that reported by Wang et al. [15]. For $\mathrm{Ra} = 10^7$, to date there are no numerical or experimental solutions in the literature which can be used to compare with this work.

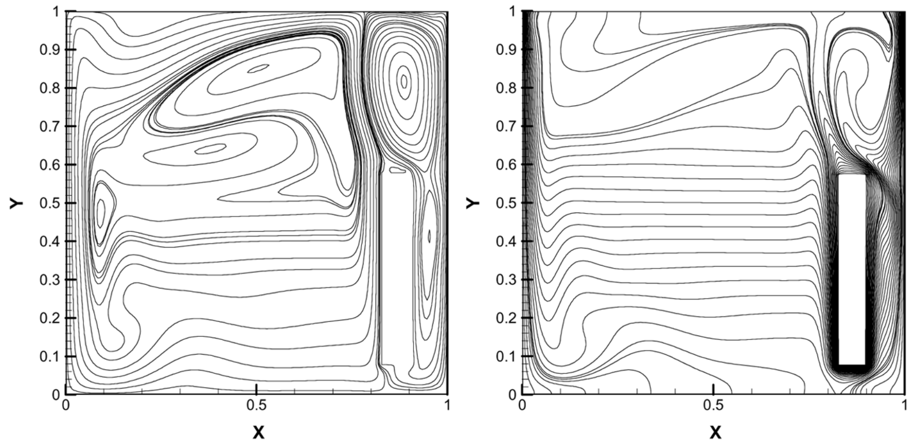
For the low-Ra, coarse-mesh flow cases, the convergence rate and stability of both the IDEAL and SIMPLER algorithms are compared. Figure 19 shows a plot of the ratios of CPU time and iteration numbers versus Ra for different grid numbers. It can be seen that the ratios decrease with increase of Ra or grid number. The ratio of iteration numbers varies from 0.138 to 0.337 and the ratio of the CPU time from 0.252 to 0.639. Figure 20 shows the convergence history, from which it can be seen that the IDEAL algorithm is much more stable than SIMPLER.

Table 6. Convergence characteristics of both SIMPLER and IDEAL for Problem 4 (C, convergence; D, divergence)

			Ra			
Grid number	Comparison terms		10^4	10^5	10^6	10^7
82 × 82	Time (s)	IDEAL	42.26	36.48	25.43	C
		SIMPLER	66.13	79.7	88.53	D
		Ratio	**0.639**	**0.458**	**0.287**	**0**
	Iteration number	IDEAL	964	837	583	C
		SIMPLER	2857	3397	3748	D
		Ratio	**0.337**	**0.246**	**0.156**	**0**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	4, 4
122 × 122	Time (s)	IDEAL	154.53	143.72	98.43	C
		SIMPLER	324.48	370.5	391.37	D
		Ratio	**0.476**	**0.388**	**0.252**	**0**
	Iteration number	IDEAL	1520	1412	964	C
		SIMPLER	5927	6657	6990	D
		Ratio	**0.256**	**0.212**	**0.138**	**0**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	4, 4
162 × 162	Time (s)	IDEAL	C	C	C	C
		SIMPLER	D	D	D	D
		Ratio	**0**	**0**	**0**	**0**
	Iteration number	IDEAL	C	C	C	C
		SIMPLER	D	D	D	D
		Ratio	**0**	**0**	**0**	**0**
	N1, N2 used in IDEAL		4, 4	4, 4	4, 4	4, 4



(*a*)



(*b*)

Figure 18. Streamline (left) and isotherm (right) patterns using the IDEAL algorithm for Problem 4: (*a*) $\mathrm{Ra} = 10^4$, grid number $= 122 \times 122$; (*b*) $\mathrm{Ra} = 10^7$, grid number $= 162 \times 162$.

It should be noted that in the four examples given for the high-Re/Ra, fine-mesh flow cases, even if the underrelaxation factors are adjusted, the SIMPLER algorithm still cannot converge.

For the four examples studied, the ratio of iteration numbers is about twice smaller than that of the CPU time. Hence we can estimate that for each iteration level the IDEAL algorithm requires about twice the computational time as the SIMPLER algorithm.

4. CONCLUSION

In this article, comprehensive comparisons have been made between the SIMPLER and IDEAL algorithms for four 2-D incompressible fluid flow and heat transfer

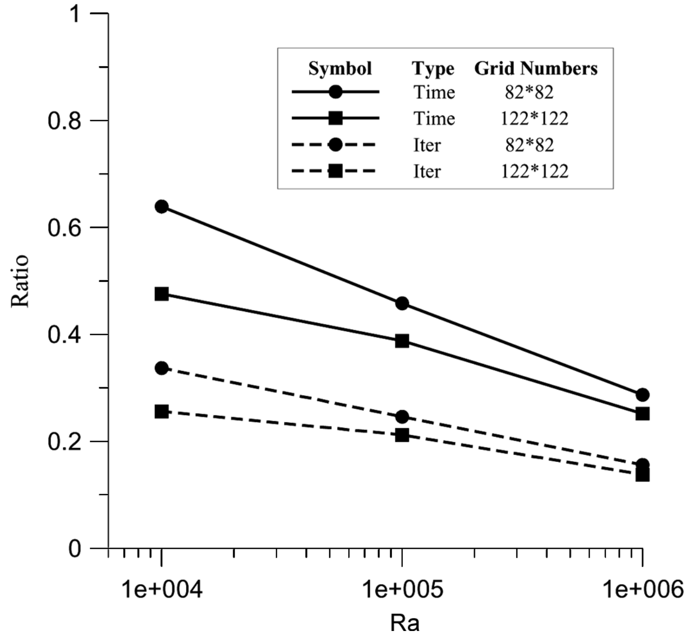


Figure 19. Ratios of CPU time and iteration numbers for Problem 4.

problems with available numerical solutions. The main conclusions are as follows.

1. The IDEAL algorithm can converge for both the low-Re/Ra, coarse-mesh and the high-Re/Ra, fine-mesh flow cases, however, the SIMPLER algorithm can converge only for the low-Re/Ra, coarse-mesh flow cases, showing very good robustness of the IDEAL algorithm.

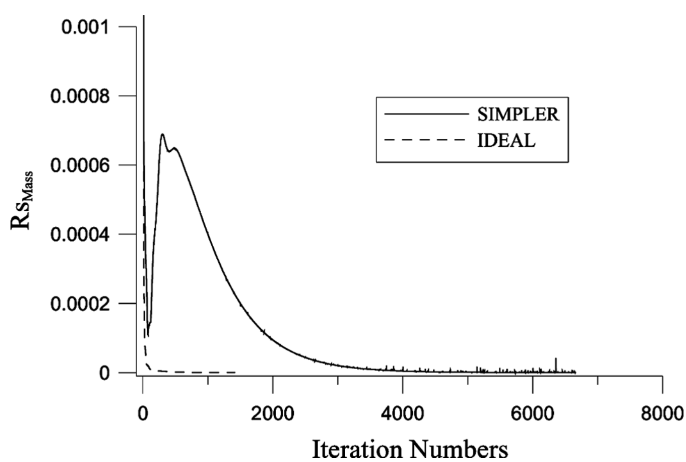


Figure 20. Convergence history of both the IDEAL and SIMPLER algorithms for $Ra = 10^5$, grid number $= 122 \times 122$ for Problem 4.

2. For low-Re/Ra, coarse-mesh flow cases, the IDEAL algorithm can significantly enhance the convergence rate and stability of the iteration process compared with the SIMPLER algorithm. For the four problems tested, the IDEAL algorithm can reduce the iteration numbers by 63.2–98.6% and the CPU time by 30.0–97.1%. Generally, the ratios of the iteration numbers and CPU time of IDEAL over that of SIMPLER decrease with increase of Re/Ra or grid number.
3. For high-Re/Ra, fine-mesh flow cases, the IDEAL algorithm can obtain converged solutions and the iteration process remains very stable, while the SIMPLER algorithm often diverges, even though the underrelaxation factors are adjusted.

Extensions to a collocated grid system, to compressible fluid flow, and to multiphase flow cases are now underway in the authors' group.

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